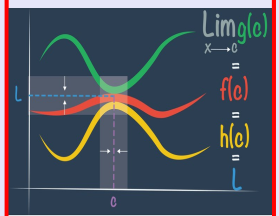
Calculus I

Lecture 26



Feb 19-8:47 AM

Class Quiz 11

Find equation of the tan. line to the graph of

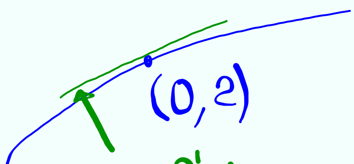
$\boxed{f(x) = \sin(x^2+x) + 2}$ at $\boxed{x=0}$

Final Ans. in Slope-Int Form

$f'(x) = \cos(x^2+x)\cdot(2x+1) + 0$

$f'(x) = (2x+1)\cos(x^2+x)$

$f'(0) = 1 \cdot \cos 0 = 1$



$m = f'(0) = 1$

$y - 2 = 1(x - 0)$

$\boxed{y = x + 2}$

Oct 14-7:07 AM

find $\dfrac{d^2y}{dx^2}$

1) $y = (x^2 - 4x + 1)^3$

$$\frac{dy}{dx} = 3(x^2-4x+1)^2 \cdot (2x-4) = 6(x-2)(x^2-4x+1)^2$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[6(x-2)(x^2-4x+1)^2\right]$$

$$= 6\left[1(x^2-4x+1)^2 + (x-2)\cdot 2(x^2-4x+1)^1 \cdot (2x-4)\right]$$

$$= 6\left[(x^2-4x+1)^2 + 4(x-2)^2(x^2-4x+1)\right]$$

$$= 6\left[(x^2-4x+1)(x^2-4x+1 + 4(x-2)^2)\right]$$

$$= 6(x^2-4x+1)(x^2-4x+1+4x^2-16x+16)$$

$$\boxed{\frac{d^2y}{dx^2} = 6(x^2-4x+1)(5x^2-20x+17)}$$

Oct 14-7:39 AM

Find $f''(x)$ for $f(x) = \left(\dfrac{2x-3}{3x-2}\right)^4$

$$f'(x) = 4\left(\frac{2x-3}{3x-2}\right)^3 \cdot \frac{2(3x-2)-(2x-3)\cdot 3}{(3x-2)^2}$$

$$f'(x) = 4\left(\frac{2x-3}{3x-2}\right)^3 \cdot \frac{5}{(3x-2)^2} \qquad f'(x) = \frac{20(2x-3)^3}{(3x-2)^5}$$

$$f''(x) = 20 \cdot \frac{3(2x-3)^2 \cdot 2 \cdot (3x-2)^5 - (2x-3)^3 \cdot 5(3x-2)^4 \cdot 3}{\left[(3x-2)^5\right]^2}$$

$$= 20 \cdot \frac{3 \cdot (2x-3)^2(3x-2)^4\left[2(3x-2) - 5(2x-3)\right]}{(3x-2)^{10}}$$

$$= \frac{60(2x-3)^2(3x-2)^{\cancel{4}}(-4x+11)}{(3x-2)^{\cancel{10}\,6}}$$

$$f''(x) = \frac{60(2x-3)^2(-4x+11)}{(3x-2)^6}$$

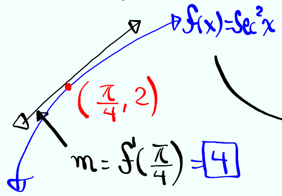
Oct 14-7:46 AM

Find $f'(x)$ for $f(x) = \tan(\underbrace{\sin^2 x + \cos^2 x}_{1})$

$f(x) = \tan 1 \rightarrow$ Constant

$\boxed{f'(x) = 0}$

Find $\boxed{\text{slope of the tan. line}}$ to the graph of $f(x) = \sec^2 x$ at $x = \frac{\pi}{4}$.



$m = f'(\frac{\pi}{4}) = \boxed{4}$

$f(\frac{\pi}{4}) = \sec^2\frac{\pi}{4}$

$= (\sqrt{2})^2 = 2$

$f(x) = [\sec x]^2$

$f'(x) = 2(\sec x)^1 \cdot \sec x \tan x$

$f'(x) = 2\sec^2 x \tan x$

$f'(\frac{\pi}{4}) = 2\sec^2\frac{\pi}{4} \cdot \tan\frac{\pi}{4}$

$= 2 \cdot 2 \cdot 1$

$= 4$

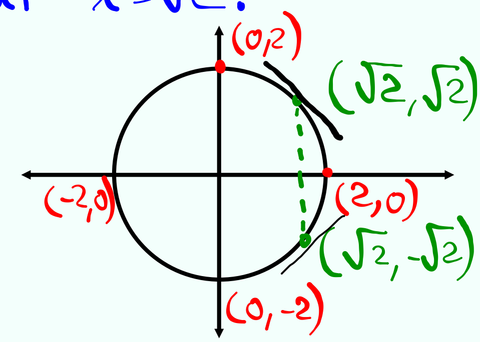
Oct 14-7:59 AM

Find $\boxed{\text{slope of the tan. line}}$ to the graph $x^2 + y^2 = 4$ at $x = \sqrt{2}$.

Circle

Center $(0,0)$

Radius 2



$(\sqrt{2})^2 + y^2 = 4$

$2 + y^2 = 4$

$y^2 = 2$

$y = \pm\sqrt{2}$

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

$y = \sqrt{4 - x^2}$ upper-half

$y = -\sqrt{4 - x^2}$ Lower-half

Oct 14-8:10 AM

Looking at the upper-half

$y = \sqrt{4-x^2}$ $\quad y = (4-x^2)^{1/2}$

$\frac{1}{2}-1 \to \frac{-1}{2}$

$y' = \frac{1}{\cancel{2}}(4-x^2)^{\frac{1}{2}-1} \cdot (-\cancel{2}x)$

$y' = \dfrac{-x}{\sqrt{4-x^2}} = \dfrac{-x}{y}$

$m = \dfrac{dy}{dx}\Big|_{(\sqrt{2},\sqrt{2})} = \dfrac{-\sqrt{2}}{\sqrt{2}} = \boxed{-1}$

Now the lower-half

$y = -\sqrt{4-x^2}$ $\quad y = -(4-x^2)^{1/2}$

$y' = -\frac{1}{\cancel{2}}(4-x^2)^{-\frac{1}{2}} \cdot (-\cancel{2}x)$

$m = \dfrac{dy}{dx}\Big|_{(\sqrt{2},-\sqrt{2})} = \dfrac{\sqrt{2}}{-(-\sqrt{2})} = \boxed{1}$

$y' = \dfrac{x}{\sqrt{4-x^2}} = \dfrac{x}{-y}$

Oct 14-8:16 AM

Implicit Diff.:

$x^2 + y^2 = 4$ $\qquad y = f(x)$

$\dfrac{d}{dx}[x^2 + y^2] = \dfrac{d}{dx}[4]$

$\dfrac{d}{dx}[x^2] + \dfrac{d}{dx}[y^2] = 0$

$2x + 2y \cdot \dfrac{dy}{dx} = 0$

$2y\dfrac{dy}{dx} = -2x$

$\dfrac{dy}{dx} = \dfrac{-\cancel{2}x}{\cancel{2}y} = \dfrac{-x}{y}$

Oct 14-8:22 AM

$$x^3 + y^3 = 2x$$

$$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 2$$

$$3y^2 \frac{dy}{dx} = 2 - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{2-3x^2}{3y^2}}$$

Find $\frac{dy}{dx}$

$$\sin \boxed{y} = \cos x^2$$

$$\cos y \cdot \frac{dy}{dx} = -\sin x^2 \cdot 2x$$

$$\frac{dy}{dx} = \frac{-2x \sin x^2}{\cos y}$$

Oct 14-8:26 AM

Find $\frac{dy}{dx}$

$$x^3 + y^4 = xy$$

$$\frac{d}{dx}[x^3] + \frac{d}{dx}[y^4] = \frac{d}{dx}[xy]$$

$$3x^2 + 4y^3 \cdot \frac{dy}{dx} = 1 \cdot y + x \cdot \frac{dy}{dx}$$

$$4y^3 \frac{dy}{dx} - x \frac{dy}{dx} = y - 3x^2$$

$$(4y^3 - x)\frac{dy}{dx} = y - 3x^2$$

$$\boxed{\frac{dy}{dx} = \frac{y - 3x^2}{4y^3 - x}}$$

Oct 14-8:30 AM